THE ANALYSIS OF ECCENTRICALLY LOADED MASONRY WALLS BY THE MOMENT MAGNIFIER METHOD

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ABSTRACT

The moment magnifier method for analysing beam-columns has been exten-sively applied to steel and reinforced concrete structures. This study concerns the application of the method to eccentrically loaded plain and reinforced masonry walls.

In this study the strength of a cross-section is based on a linear stress diagram which takes into account the tensile strength of the masonry. The critical buckling load is based on the properties of the wall accounting for the extent of crack penetration and the portion of wall height subjected to cracking.

For a given eccentricity condition, the ultimate load capacity of a wall is determined by an iterative procedure. The first step is the calcula-tion of the capacity assuming a short wall. The critical buckling load is then calculated and a new value of the ultimate load is determined by applying the moment magnifier equation. The procedure is repeated until the load converges.

INTRODUCTION

Masonry walls are frequently loaded eccentrically and must be analysed for the effects of combined bending and axial load. The behavior of such walls is affected by material properties, the presence of reinforce-ment, flexural stiffness and column slenderness. An evaluation of the ultimate capacity must recognize the increase in applied moments due to the so-called P-D effect. The moment magnifier method provides a means of accounting for this effect.

The relationship between axial load and moment may be conveniently des-cribed by interaction diagrams. Such diagrams may be constructed for different cases of end eccentricities and for walls of different slenderness values.

This present study describes an analysis of masonry walls based on the moment magnifier method of assessing the effects of combined bending and axial load. The analysis accounts for the effects of material pro-perties, wall slenderness, the presence of cracks and the presence of reinforcement. A method of developing interaction diagrams for various conditions of end eccentricity is presented.

Analysis for Combined Bending And Axial Load

Consider a structural member subjected to eccentric loading as shown in Fig. 1(a). Under the action of the equal end moments, the member will deform as shown in Figure 1(b). On the basis of elastic behavior the basic differential equation is

$$EI \frac{d^4v}{dz^4} + P \frac{d^2v}{dz^2} = 0 \qquad (1)$$

Letting $k = \sqrt{P/EI}$, the lateral displacement

$$v = e\left\{\left(\frac{1-\cos kh}{\sin kh}\right) \sin kz + \cos kz - 1\right\} \qquad (2)$$

The maximum value of v occurs at mid-height as shown in Figure 1(c) and may be expressed to a satisfactory degree of accuracy as

$$v_{max} = \frac{\bar{v}}{(1 - P/P_{cr})} \qquad (3)$$

where $\bar{v}$ is the lateral deflection at mid-height due to the moment P_e only and P_{cr} is the critical buckling load.

The same increase from $\bar{v}$ to v_{max} would result if the applied moment P_e was increased to

$$M = Pe \frac{v_{max}}{\bar{v}} \qquad (4)$$

or

$$M = \frac{Pe}{(1 - P/P_{cr})} \qquad (5)$$

The term $1/(1-P/P_{cr})$ is defined as the magnification factor and M is the magnified moment. Conditions at mid-height may therefore be analysed for the axial load P and the magnified moment M.

Other end moment conditions result in different magnification factors; however all conditions may be referenced to the case of equal end moments. For any end condition

$$M = Pe \frac{C_m}{(1 - P/P_{cr})}$$

where C_m is a factor which converts the given moment condition to an equivalent equal end moment condition.

Evaluation of Pcr for Masonry Walls

The value of P_{cr} is required for the evaluation of the magnification factor. Its value is affected by cracks which develop in the wall due to the lateral deflection produced by end moments. The cracks produce a reduction in flexural stiffness EI which is proportional to the eccentricity of the applied load.

Yokel (1) developed the differential equations for the buckling of plain masonry walls in which the masonry has no tensile strength, i.e. cracks penetrate on the tension side to the point where the stress is equal to zero. The solution of these equations for a solid wall gives

$$P_{cr} = 0.641\,\pi^2\,(0.5 - e/t)^3\,\frac{Ebt^3}{h^2} \qquad (6)$$

where
- e = eccentricity of vertical load
- t = thickness
- b = width
- h = height
- E = modulus of elasticity

Letting $I_o = \frac{bt^3}{12}$ = moment of inertia of the full cross-section

$$P_{cr} = 7.69\,\pi^2\,(0.5 - e/t)^3\,\frac{EI_o}{h^2} \qquad (7)$$

In this form the equation may be applied to solid or hollow sections. The equation may be approximated as

$$P_{cr} = 8\pi^2\,(0.5 - e/t)^3\,\frac{EI_o}{h^2} \qquad (8)$$

This equation compares favorably with the solutions presented by Chapman and Slatford (2).

Letting $I_1 \;=\; 8(0.5 - e/t)^3\; I_o$ (9)

where I_1 = the moment of inertia of the uncracked portion of the cross-section.

the critical buckling load can then be expressed as

$$P_{cr} = \frac{\pi^2 E I_1}{h^2} \tag{10}$$

If we consider that the masonry possesses some tensile strength f'_t the conditions on the cross-section will correspond to Figure 2. Cracks penetrate to the point where the tension stress equals f'_t and

$$I_1 = 8[0.5 - e/t + \frac{\zeta}{2t}]^3\; I_o \tag{11}$$

where ζ is the distance from the point of zero stress to the end of the crack. This distance is obtained from the following relations

$$f_{max} = \frac{4}{3}\left[\frac{P}{A(1 - \frac{2e}{t})}\right] \tag{12}$$

$$\xi = \frac{2tP}{Af_{max}} \tag{13}$$

$$\zeta = \frac{\xi f'_t}{f_{max}} \tag{14}$$

If the wall is subjected to unequal end eccentricities, e_1 and e_2, producing single curvature bending, the value of e in the above equations may be taken as equal to the average of e_1 and e_2. Although this is an approximation, it gives satisfactory results.

A method for determining the buckling load for the case of double curvature bending has been developed by the writers (3). Even though walls bent in double curvature may be expected to crack as shown in Figure 3a, in fact buckling will tend to occur in the primary single loop con-figuration shown in Figure 3b. This behavior is substantiated in tests as indicated by typical results shown in Figure 4. The properties of the wall therefore may be approximated as a "stepped" column as shown in Figure 5. The critical buckling load may be expressed as

$$P_{cr} = \lambda \frac{EI_o}{h^2} \qquad (15)$$

where λ is a buckling coefficient for the "stepped" column which depends on I_1/I_o and $e_1/(e_1+e_2)$. A complete table of λ values is presented in Reference (3).

The buckling load for a reinforced masonry wall in single curvature can be evaluated by Equation (10) for eccentricities smaller than t/3. For larger eccentricities the reinforcement will be in tension and the behavior will be related to the moment of inertia of the transformed section. A lower limit may be placed on the flexural stiffness as proposed by MacGregor et al. (4) for reinforced concrete, namely

$$EI_o\ [0.5 - e/t] \geq 0.10\ EI_o \qquad (16)$$

The analysis of reinforced walls in double curvature may be carried out similarly to the analysis of plain masonry walls described above.

Interaction Diagrams

An interaction diagram for a masonry wall is dependent on the wall thickness, the height of wall, the end eccentricities and the material properties. In the experimental phase of this study wall test specimens were 40 inches wide and composed of nominal 8-inch hollow concrete blocks (t = 7-5/8"). Reinforced walls contained either three #3 bars or three #9 bars. The interaction diagrams presented in this paper conform to these properties. The following material properties were used, based on the results of tests:

f'_m = 2500 psi

f'_t = 200 psi

E = 1.125×10^6 psi

A computer program was developed to construct the interaction diagrams. The following assumptions were made in the analysis:

1. sections plane before loading remain plane during loading.
2. The stress strain relationship for masonry is parabolic.
3. No tensile capacity for masonry is included in the calculation for moment resistance.
4. The strain in the reinforcement is the same as in the surrounding grout.

5. The stress-strain relationship for the reinforcement is linear up to the yield stress at which point the behavior becomes perfectly plastic.

a) Plain Masonry Walls

(i) Single Curvature with Equal End Eccentricities

Case I - h/t = o

There are no slenderness effects for this case. Points on the interaction diagram may then be directly calculated for various values of e/t. The resulting interaction diagram is shown in Figure 6.

Case II - h/t > o

Slenderness effects must be taken into account for this case and therefore an iterative procedure must be employed. The interaction diagram for h/t = o is an upper bound solution and can be used to establish a starting point for the iterative procedure. The procedure is as follows:

1. For a particular e/t ratio obtain values for P and M from the interaction diagram based on h/t = 0. These values are the first approximations for the values for the particular h/t value being considered.

2. Using the value of P obtained in Step 1, calculate f_{max}, ξ and ζ by means of Equations (12), (13) and (14). Determine I_1 from Equation (11) and then calculate P_{cr} by means of Equation (10).

3. By substituting the value of M from Step 1 and the value of P_{cr} from Step 2 into Equation (5), solve for a new value of the load P.

4. Obtain the corresponding value for M from the interaction diagram for h/t = 0. This value together with the value of P obtained in Step 3 constitute the new values to be used in the second cycle of iteration.

Steps 2, 3 and 4 are repeated until convergence in the value of P occurs. Convergence will be rapid and only two or three cycles will be necessary to complete the solution.

The following numerical example illustrates the calculations for a wall with h/t = 6.5. The wall is 40 inches wide and composed of 8-inch hollow concrete blocks (t = 7-5/8"). It is desired to obtain the point on the interaction diagram for e/t = 1/3. The calculations are carried out as follows:

1. From the interaction diagram for h/t = 0 (Figure 6) p = 135 kips. The corresponding moment M = Pe = Pt/3 = 343 kip-in.

2. From Equation (12), f_{max} = 1.77 ksi
   From Equation (13), ξ = 3.8 in.
   From Equation (14), ζ = 0.43 in.
   From Equation (10), P_{cr} = 396 kips.

3. From Equation (5)

$$M = \frac{Pe}{(1 - P/P_{cr})}$$

$$343 = \frac{P \times 2.54}{(1 - P/396)}$$

$$P = 100 \text{ kips}$$

4. From the interaction diagram for h/t = 0, M = Pe = 320 kips.

The second cycle of iteration therefore begins with P = 100 kips, M = 320 kip-in. Repeating Steps 2 and 3 results in a revised value of P = 99 kips which indicates satisfactory convergence. The corresponding value of M = Pe is 252 kip-in. Therefore the point on the interaction diagram for e/t = 1/3 is P = 99 kips and M = 252 kip-in.

Other points on the interaction diagram are obtained by considering other values of e/t. Figure 6 shows the completed diagram.

Interaction diagrams for other eccentricity conditions in plain walls and for reinforced walls are constructed by means of similar procedures to those described above. Figures 7, 8, 9, 10 and 11 present interaction diagrams for these various conditions.

Experimental values are plotted for comparison with the theoretical interaction diagrams. As expected, there are significant differences in some cases. However, the experimental results are in fairly good agreement with the analytical values. Also plotted on the Figures are interaction curves derived from CSA Standard S304 "Masonry Design and Construction for Buildings" (5), These curves are based on the CSA provisions for reduction in capacity due to slenderness and eccentricity and a factor of safety of 1.0 to account for failure conditions.

Complete data for all wall tests is presented in two reports (6) and (7) prepared by the authors.

Conclusion

The moment magnifier method is an effective method of analysing ultimate failure conditions in eccentrically loaded masonry walls. The effects of cracking, reinforcement and various eccentricity conditions can be satisfactorily taken into account in the method. The results of the moment magnifier analysis compare favorably with experimental results.

Acknowledgements

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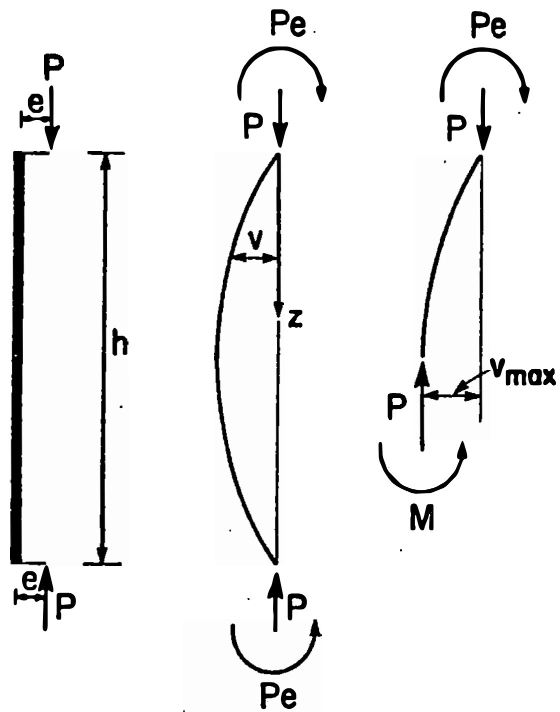

(a) (b) (c)

Figure 1 Eccentrically Loaded Member

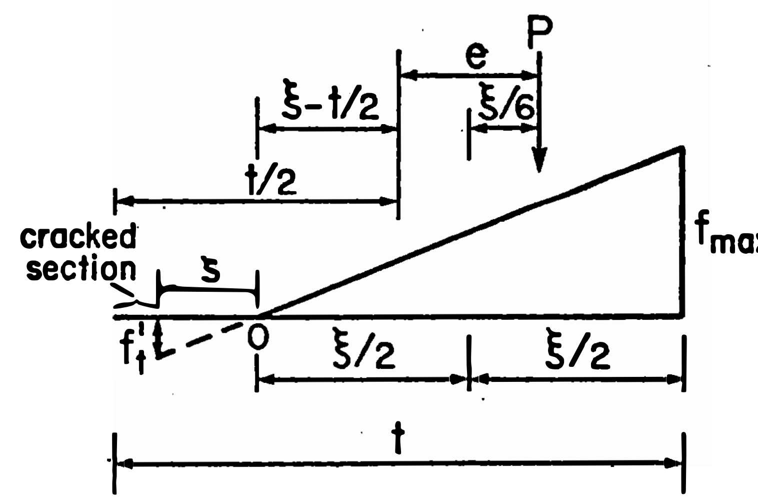

Figure 2 Stress Distribution on Masonry Section

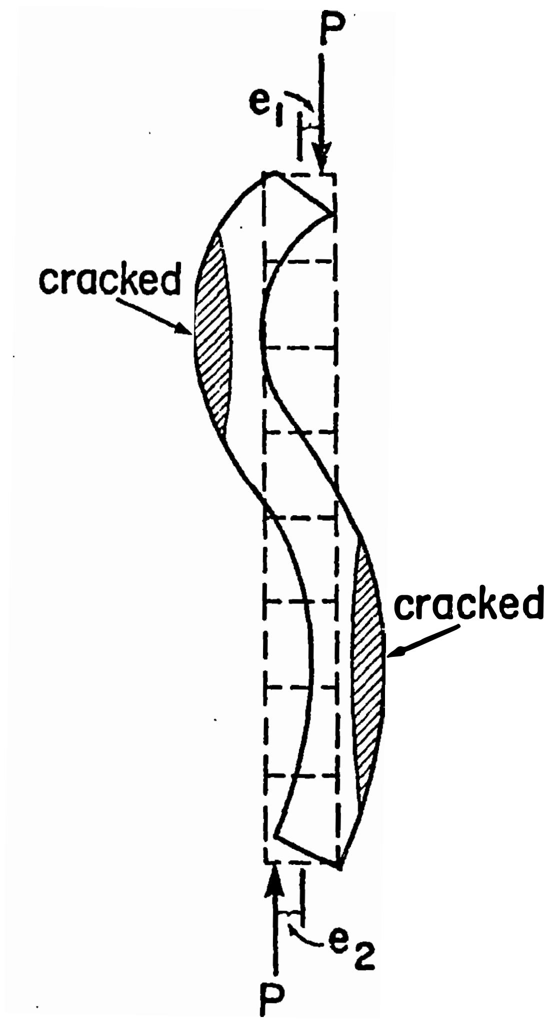

Figure 3a Wall Bent in Double Curvature

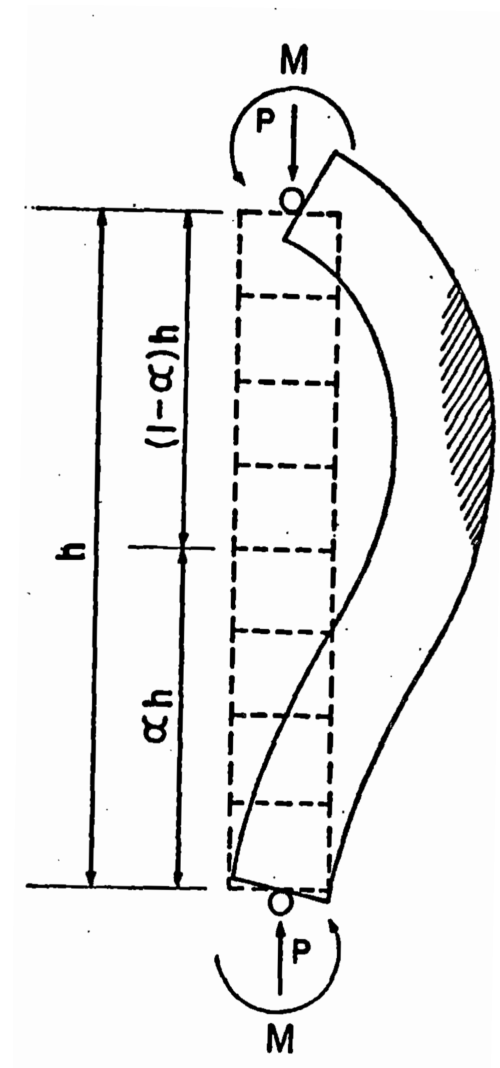

Figure 3b Buckling Mode for Wall in Double curvature Subjected to Opposite End Eccentricities

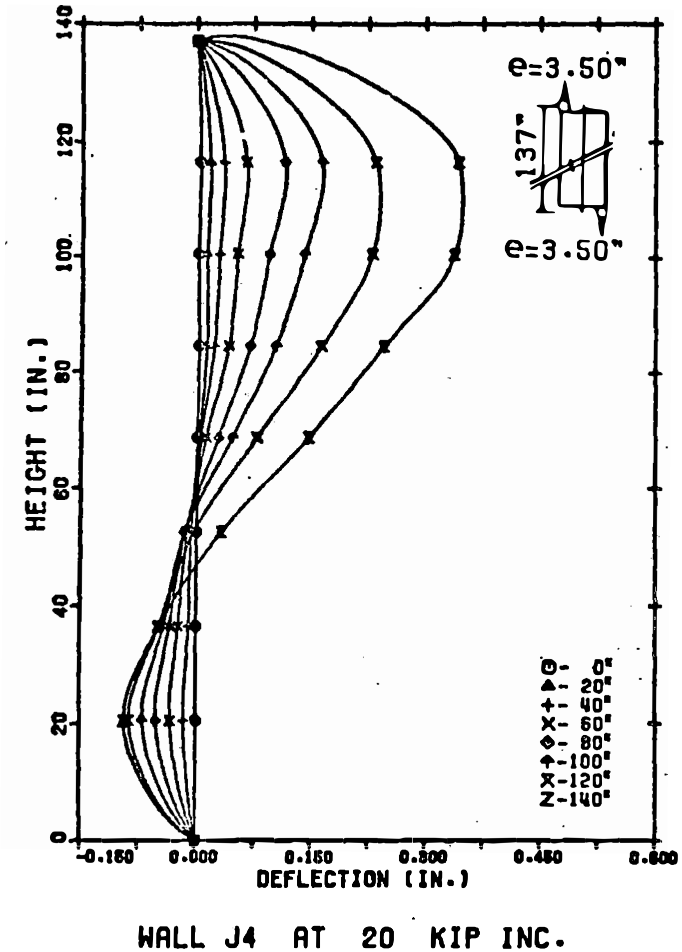

Figure 4 Typical Deflected Shape of Wall Tested with Opposite End Eccentricities

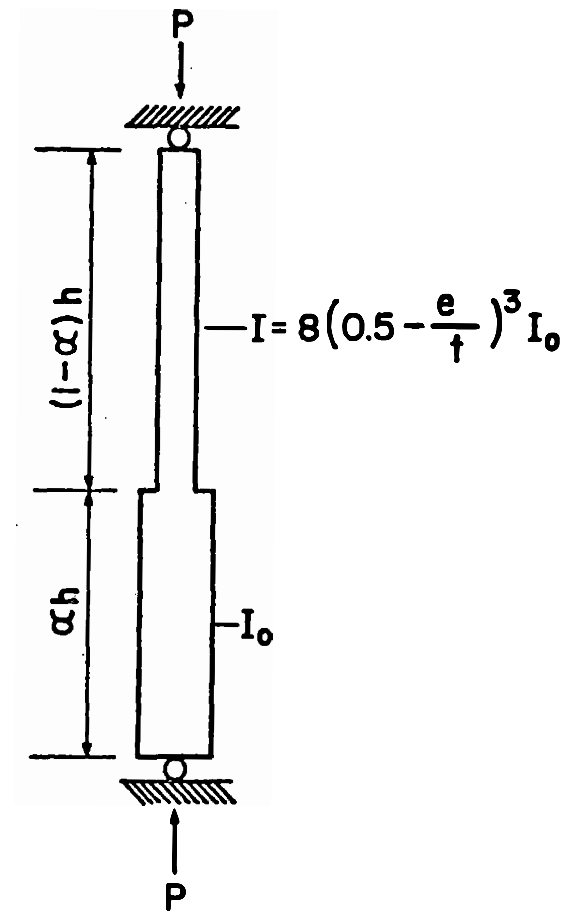

Figure 5 Equivalent Stepped Column Section

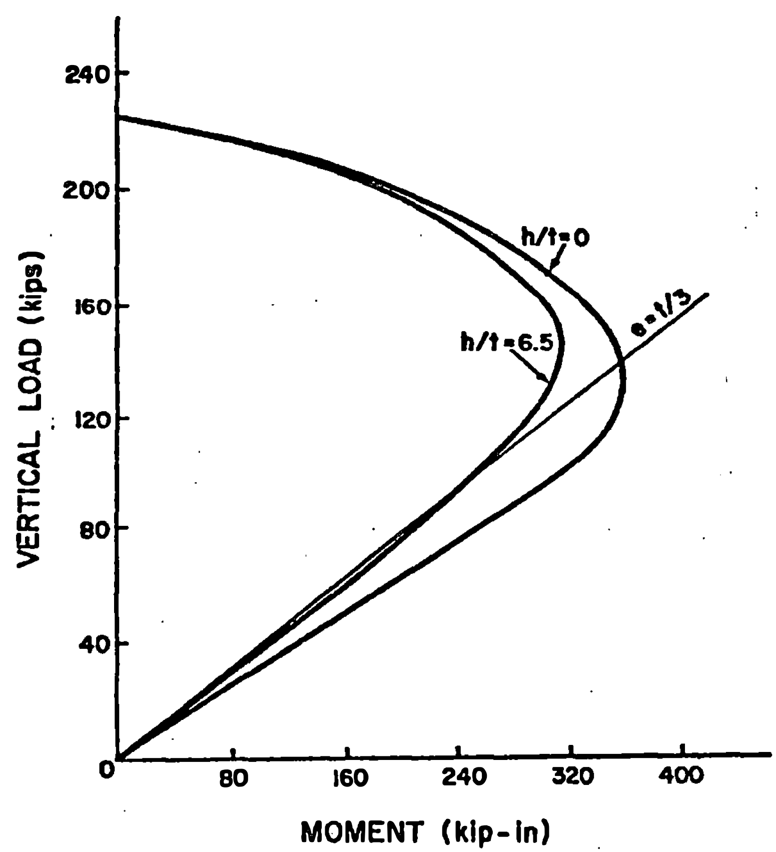

Figure 6 Effect of Slenderness on Interaction Diagram

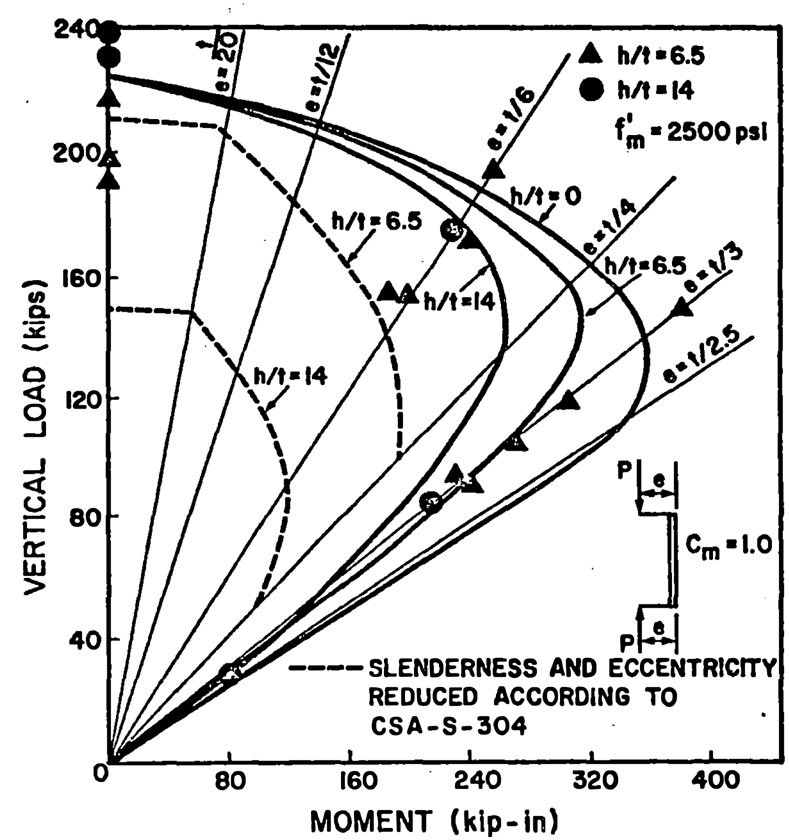

Figure 7 Interaction Diagrams for Plain Masonry Walls with Equal End Eccentricities

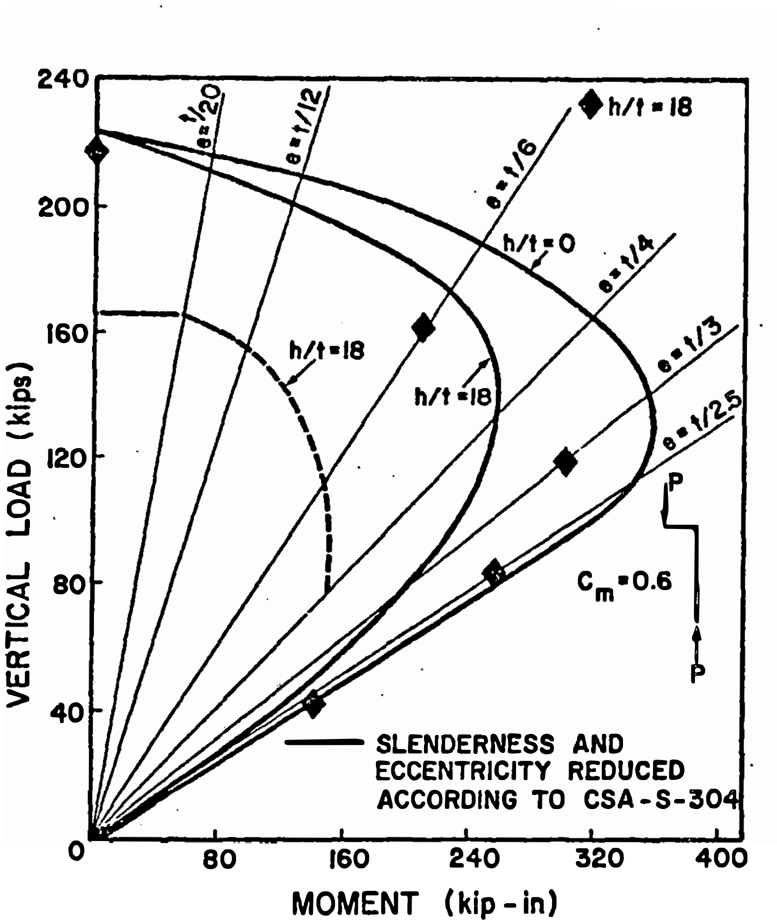

Figure 8 Interaction Diagrams for Plain Masonry Walls with Zero Eccentricity at Base

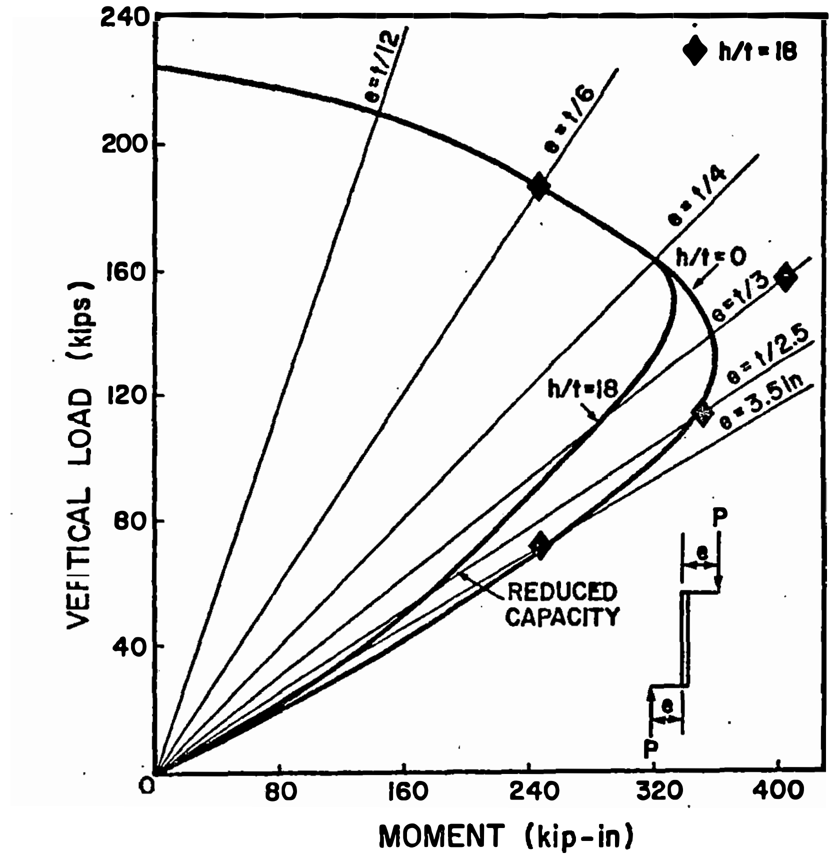

Figure 9 Interaction Diagrams for Plain Masonry Walls with Equal and Opposite End Eccentricities.

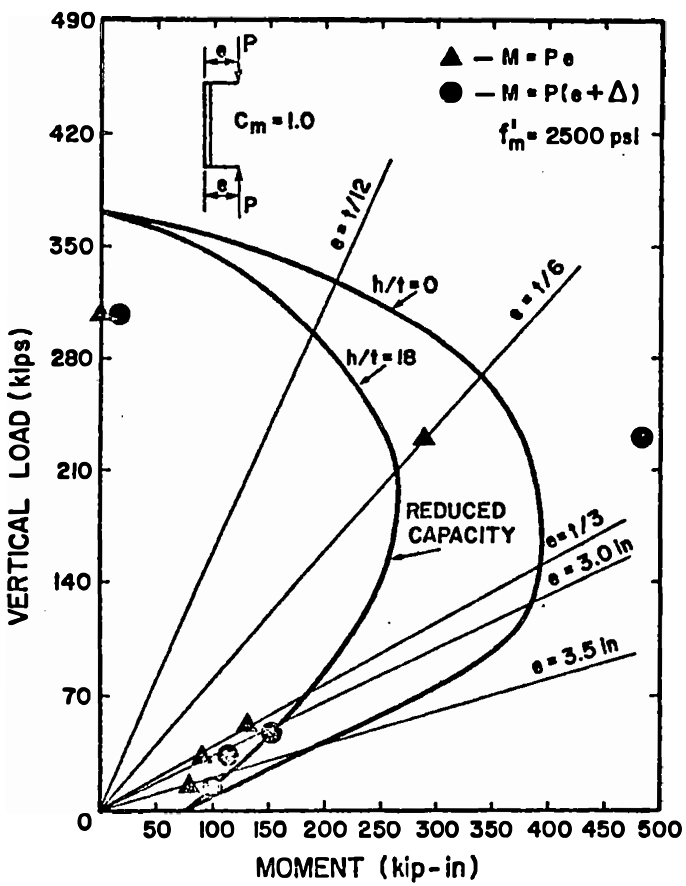

Figure 10 Interaction Diagrams for Masonry Walls Reinforced with 3-#3 Bars and with Equal End Eccentricities

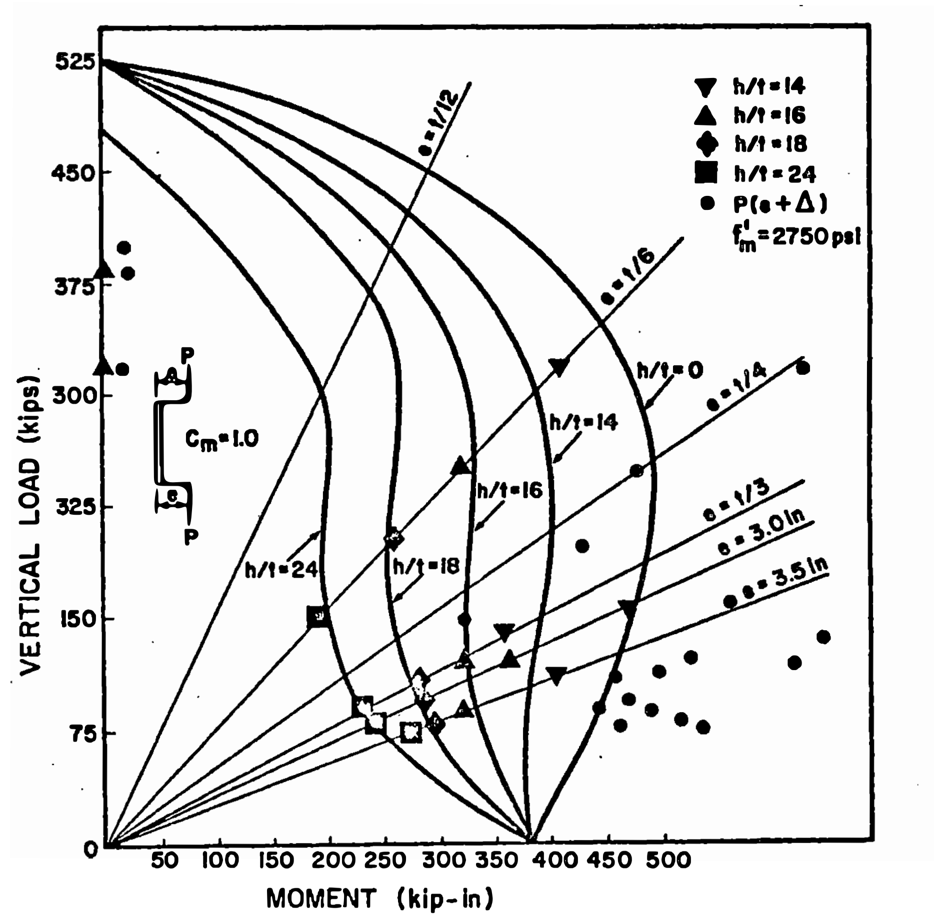

Figure 11 Interaction Diagrams for Masonry Walls Reinforced with 3-#9 Bars and with Equal End Eccentricities